

Cantilever Beam Bending Stress Validation

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Discipline: Mechanical Design & Mechanics of Materials

Tools: SOLIDWORKS, ANSYS, MATLAB, Microsoft Excel, FDM 3D Printing

Abstract

This report validates classical cantilever-beam bending theory through a combined analytical and numerical approach via Finite Element Analysis (FEA) and an experimental study using 3D-printed PLA specimens. Closed-form Euler–Bernoulli solutions were derived and implemented in MATLAB, stress distributions were verified in ANSYS, and benchtop experiments were conducted to failure. Analytical and experimental failure loads agreed within ~1% while deflection predictions exhibited larger deviations due to boundary compliance, material anisotropy, and hygroscopic effects in the PLA. The study demonstrates that stress-based failure predictions remain robust even when global stiffness predictions are sensitive to real-world effects.

1. Objective

Validate bending-stress-based failure prediction for a cantilever beam and reconcile discrepancies between analytical deflection (hand calculation, MATLAB, and FEA) and experimental response for a 3D-printed PLA component.

2. Geometry, Material, and Assumptions

2.1 Geometry

- Total printed length: 7.00 in
- Clamped length: 0.50 in
- Geometric free span: 6.50 in
- Cross-section: rectangular (width $w = 1.00$ in, thickness $t = 0.375$ in)

2.2 Polymeric Material – Polylactic Acid (PLA)

Custom material properties were defined in both SOLIDWORKS and ANSYS using effective isotropic mechanical properties derived from the additive printing manufacturer's Technical Data Sheet (TDS) provided by Bambu Lab. The TDS reports direction-dependent properties, indicating that the material is anisotropic due to the layer-wise nature of FDM 3-D printing.

For the purposes of analytical and numerical modeling, PLA was treated as a brittle material with conservative, effective isotropic properties: Young's modulus (effective isotropic) of $E \approx 290.00$ ksi and ultimate tensile strength of $\sigma_{ult} \approx 4.35$ ksi

This approach reflects common engineering practice when full anisotropic property sets are unavailable, while still acknowledging the dominant influence of Z-direction interlayer weakness and hygroscopic effects on stiffness and long-term failure behavior.

2.3 Assumptions

- Euler–Bernoulli beam theory
- Linear elastic behavior up to brittle fracture
- Bending-dominated stress state; shear negligible
- Effective isotropic properties were used due to limited anisotropic data

3. Analytical Model

3.1 Governing Stress

The extreme-fiber bending stress at the fixed end was derived symbolically using Macaulay's (singularity) function formulation, rather than immediately reducing to a canonical tip-load cantilever equation. This approach preserves a fully generic load-position representation, enabling direct extension of the formulation to alternate loading and boundary conditions.

The governing bending stress at the extreme fiber is expressed as:

$$\sigma_b = \sigma_{ult} = \frac{6PL}{wt^2}$$

Using $L = 6.5$ inch and the ultimate tensile strength of PLA, the analytically predicted failure load is:

$$P = \frac{\sigma_b wt^2}{6L} \equiv P_{fail} \approx 15.69 \text{ lbf}$$

3.2 Deflection

Deflection was derived using the Macaulay singularity function method to maintain a generalized beam equation applicable to arbitrary load locations. Although the formulation reduces to the classical cantilever tip-load expression for this specific case, it was intentionally retained in its generalized form to emphasize modeling rigor and analytical flexibility. Deflection predictions were therefore used for trend comparison rather than as the governing failure criterion.

$$\delta = \frac{1}{EI} \left(-\frac{M}{2} \langle x \rangle^2 + \frac{P}{6} \langle x \rangle^3 \right)$$

4. Finite Element Analysis (SOLIDWORKS to ANSYS)

4.1 Model Setup

The beam geometry was parametrically modeled in SOLIDWORKS and subsequently imported into ANSYS for static structural analysis. This CAD-to-CAE workflow mirrors standard industry practice by separating geometric definition from numerical analysis.

The model employed:

- Linear elastic material behavior,
- Mesh refinement near the fixed end to capture extreme-fiber stresses,
- And a concentrated load applied at the free end.

4.2 Failure Criteria

- Maximum principal stress was used as the governing criterion for brittle failure.
- Von Mises stress evaluated as a sanity check

Under the uniaxial bending-dominated stress state, the von Mises stress converges to the maximum normal stress, confirming the internal consistency of the stress field while preserving the maximum principal stress as the governing failure criterion.

Figure Set A — Deformation Max of 1.05 in

- *FEA Deformation – Deflection (Tip View)*

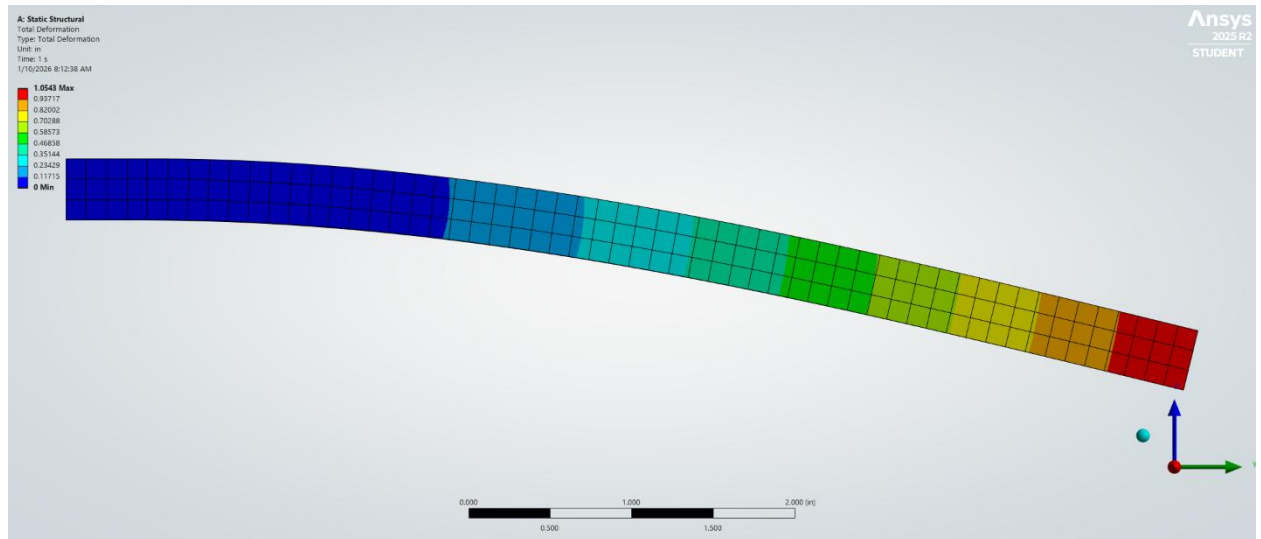
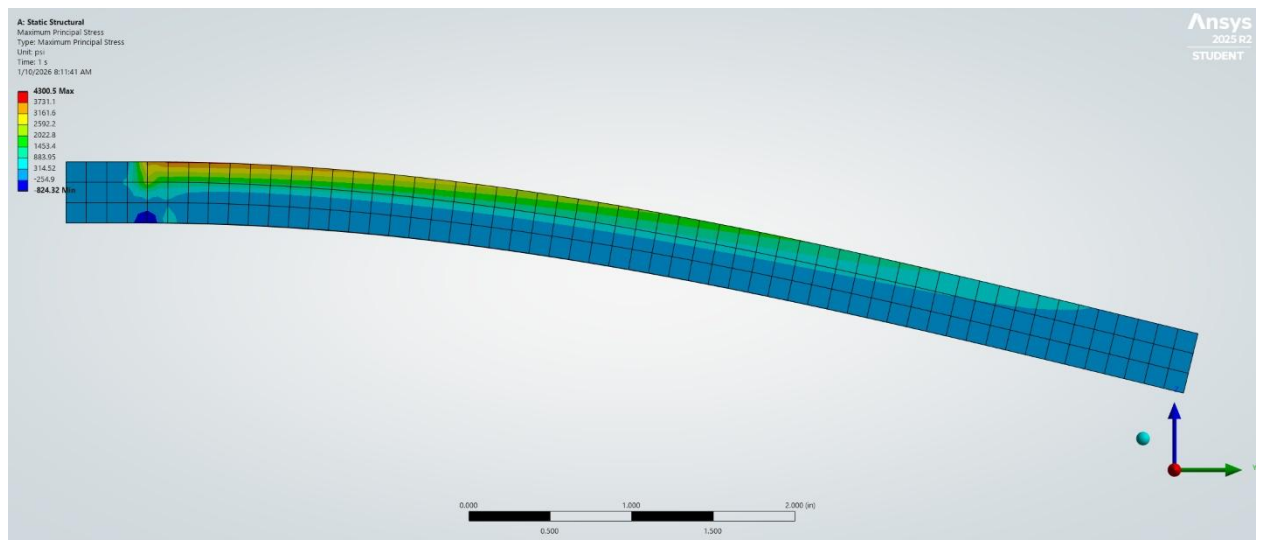


Figure Set B — Maximum Principal Stress (Brittle) of 4.30 ksi

- *FEA Front View – Max Principal Stress*



- *FEA Isometric View – Max Principal Stress*

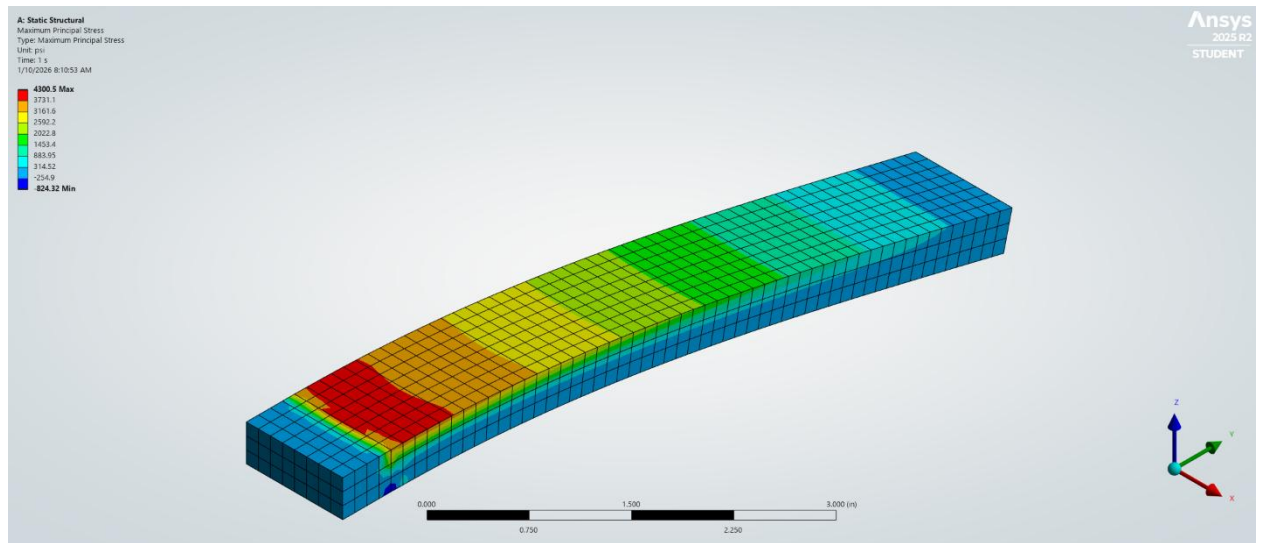
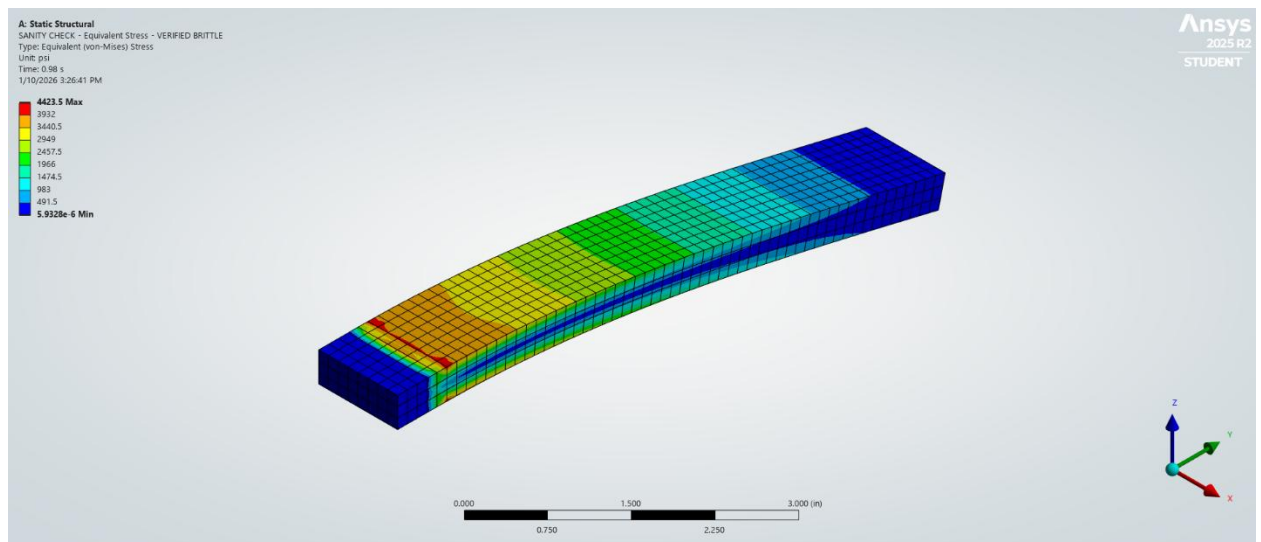


Figure Set C — Equivalent (von Mises) Stress (Sanity Check) of 4.42 ksi

- *FEA Isometric View – Equivalent (von Mises) Stress Sanity Check*



5. Experimental Methodology

5.1 Setup

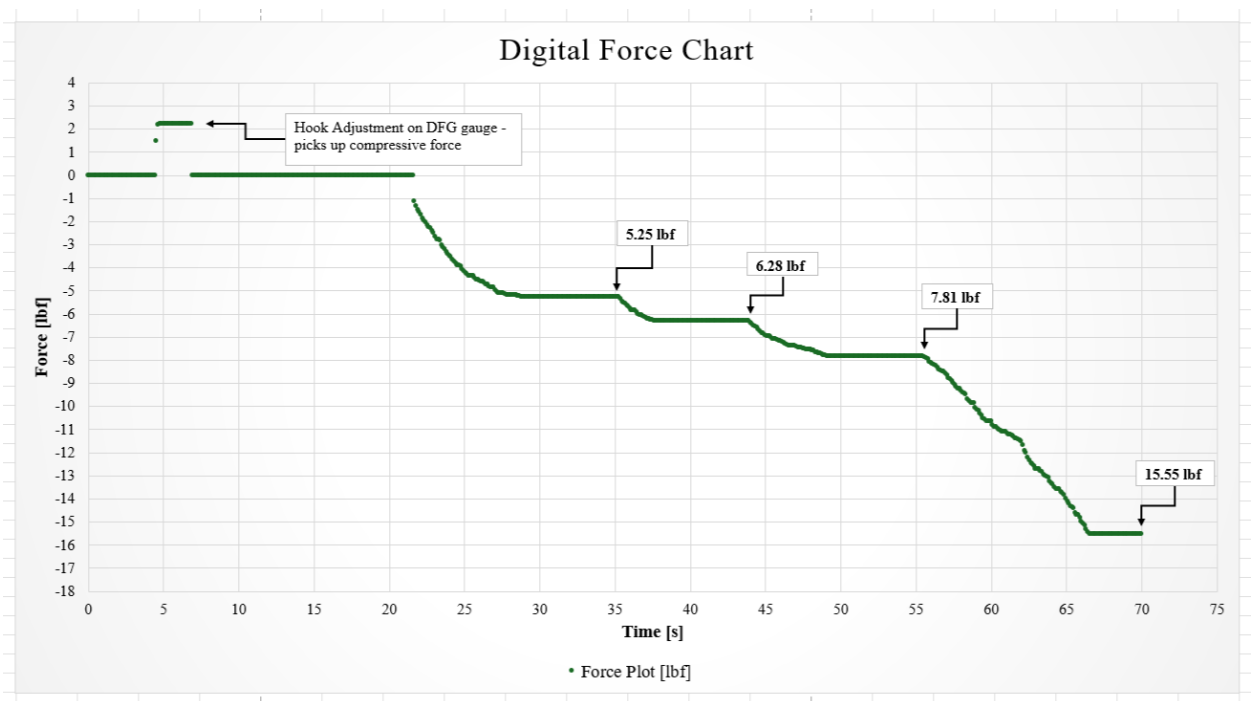
- Specimens printed with 0° raster orientation aligned to minimize off-axis effects.
- Load applied incrementally using a digital force gauge.
- And deflection measured with a drop dial indicator (1 in travel limit).

5.2 Data Acquisition

- Force–time data logged digitally.
- Stepwise loading pauses are used to record deflection.

Figure D — Digital Force Chart

- *Force vs Time with annotated load levels*



6. Results with 1.00 Factor of Safety

6.1 Failure Load

- Analytical prediction: 15.69 lbf
- Experimental (fresh prints): 15.55 lbf
- Error: ~0.89%

6.2 Deflection

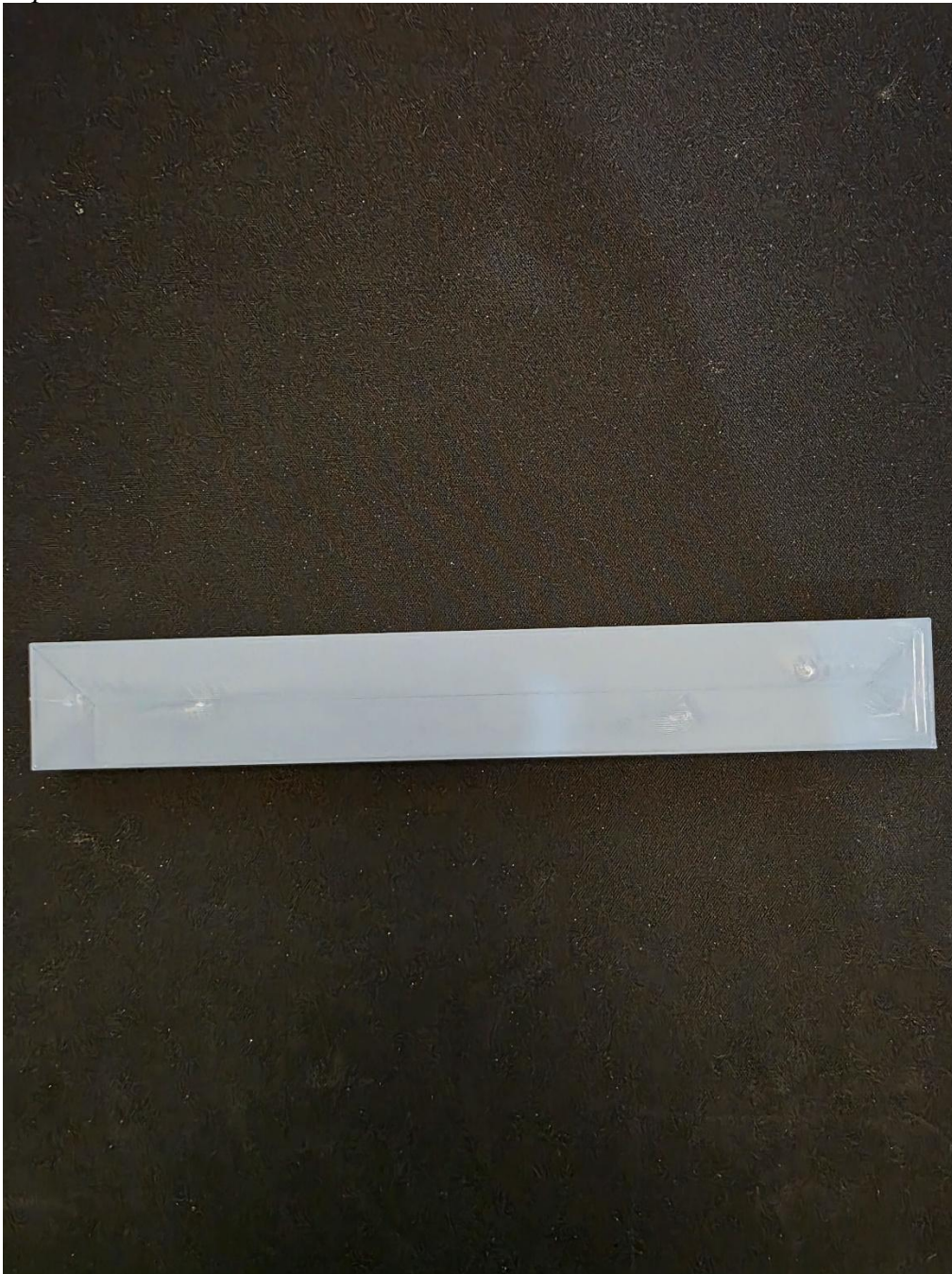
Measured deflections exceeded analytical predictions by 25–35% across load levels, due to boundary-condition realism, including clamp compliance, anisotropy, and hygroscopic effects. Deflection exceeded the dial indicator range at failure and was not recorded.

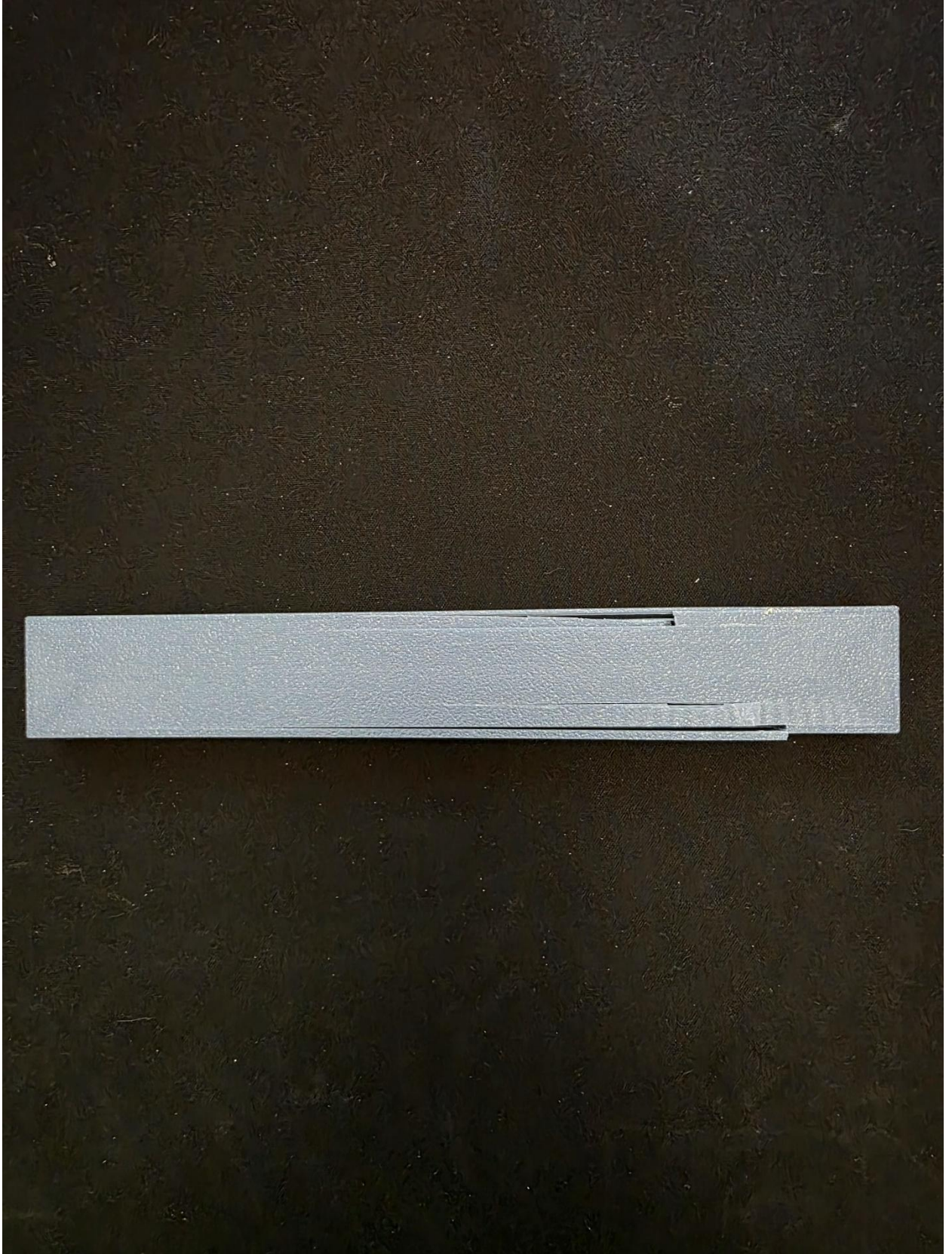
6.3 Failure Mode

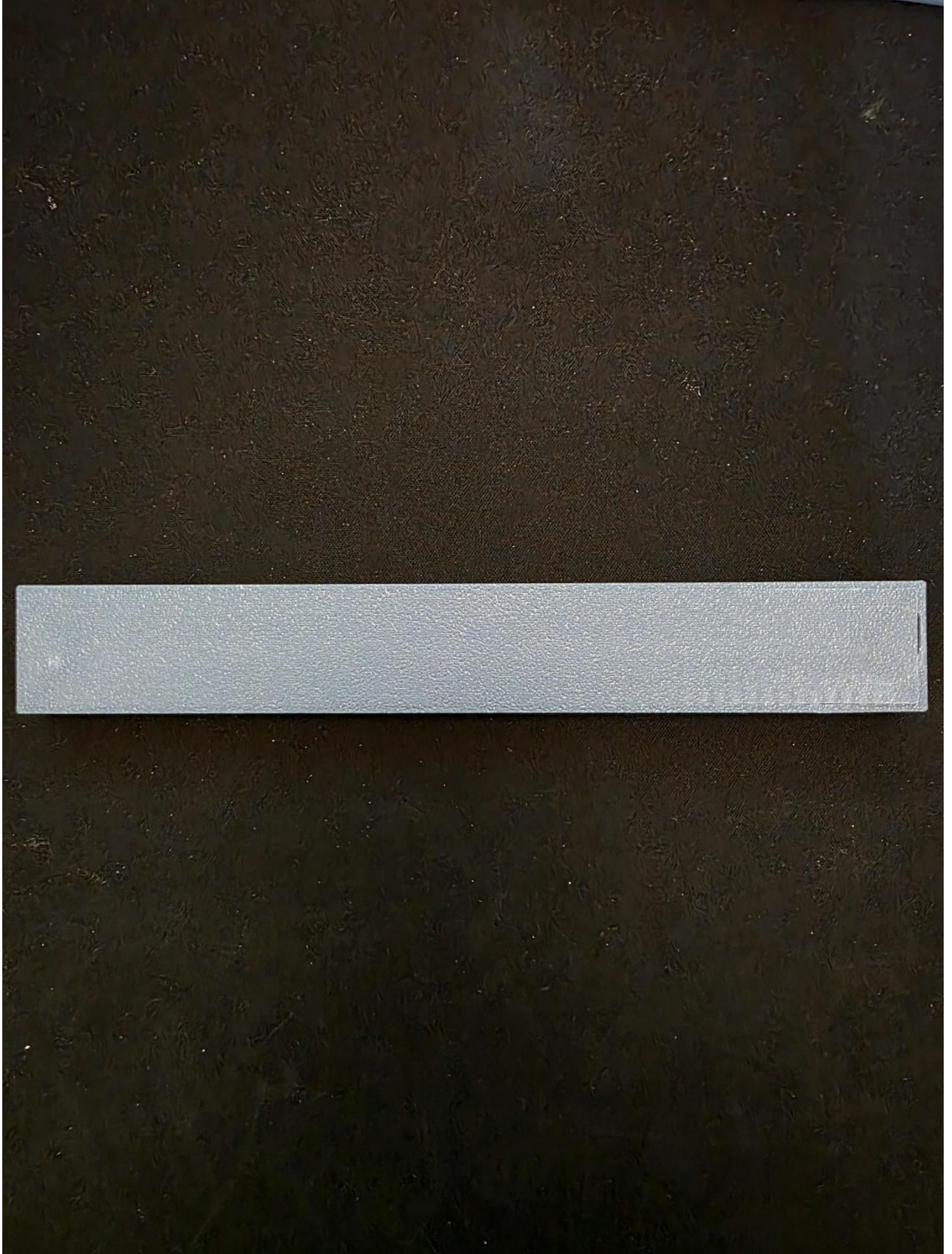
Fracture initiated at the extreme tensile fiber near the fixed end, aligned with the Z-axis ($\hat{k}$ -direction) layer interface, confirming brittle failure governed by interlayer strength.

Figure Set E — Specimen Failure (Photos)

- *Top Views:*



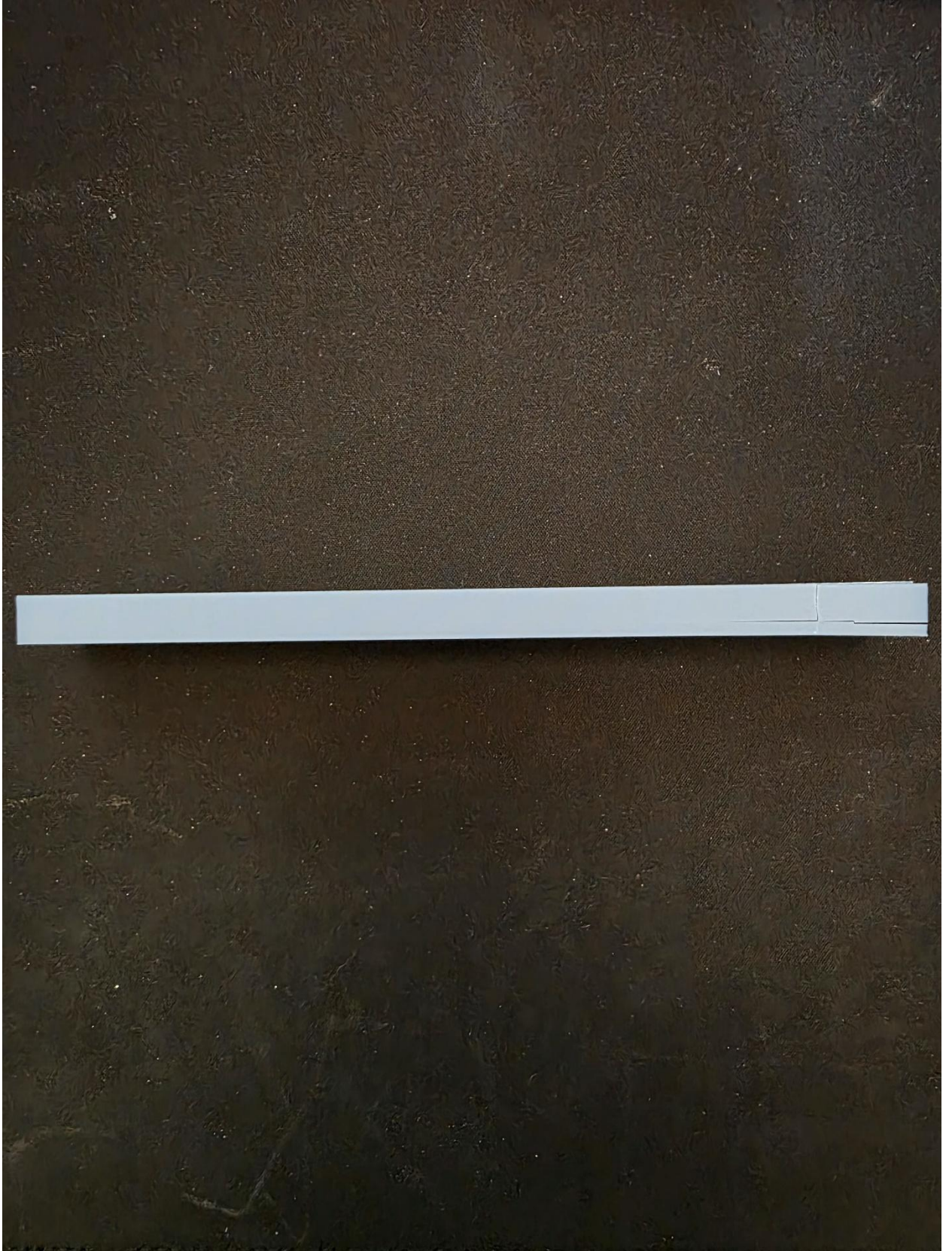


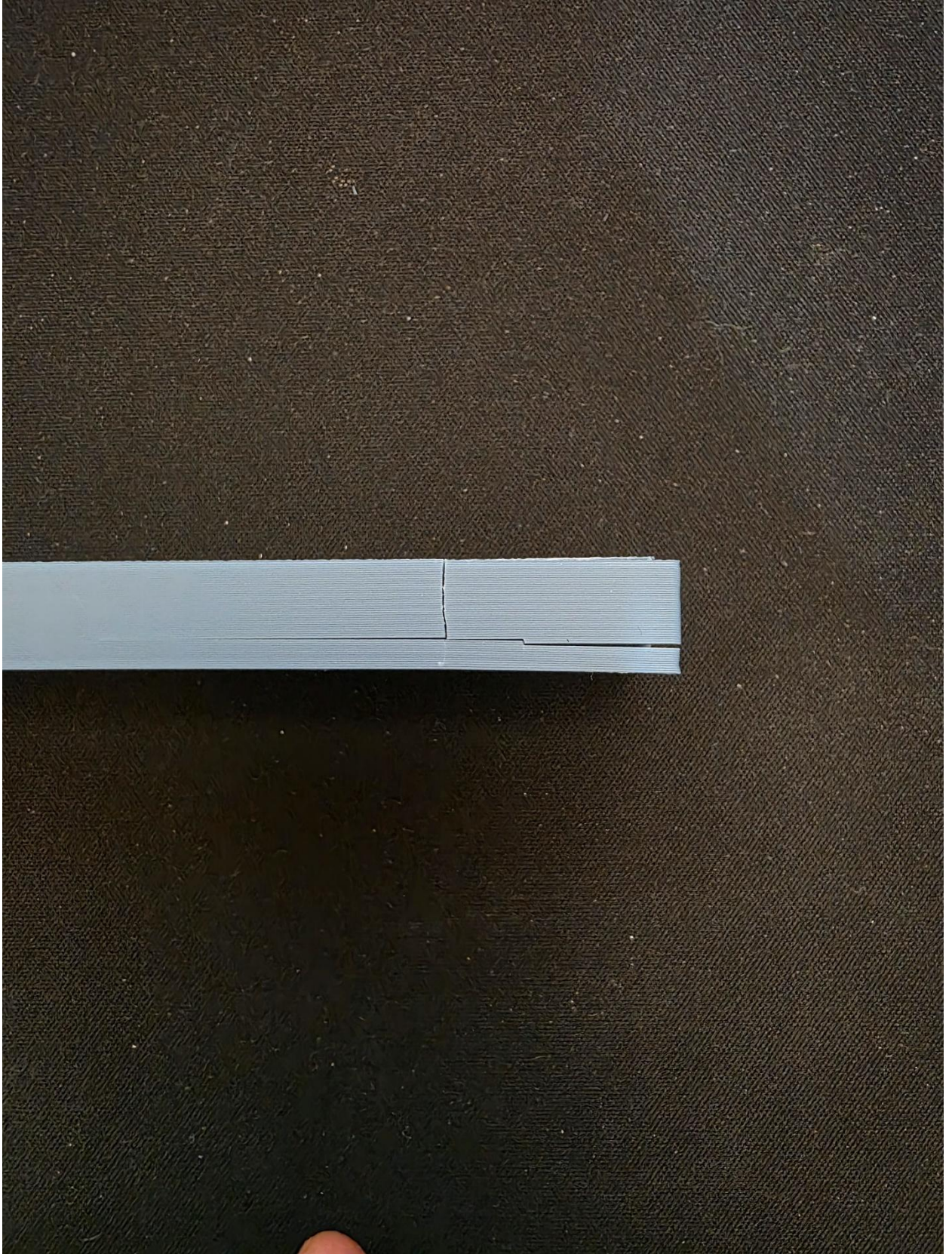


- *Front Views:*









- *Right Views:*





7. Discussion

7.1 Stress vs Deflection Accuracy

Stress-based failure predictions agreed closely across theory, FEA, and experiment, while deflection predictions were more sensitive to boundary and material effects. This demonstrates that local stress criteria are robust even when global stiffness is affected by non-ideal support and anisotropy.

7.2 Anisotropy and Hygroscopic Effects

PLA's layer-wise anisotropy and moisture absorption reduced effective stiffness and, for older specimens, strength. Fresh prints showed significantly improved agreement with theory, highlighting the importance of material conditioning.

7.3 Boundary Compliance

Clamp rotation and compliance effectively increase the active span under load, amplifying deflection without materially altering peak bending stress at the fixed end.

8. Conclusions

- Classical bending theory accurately predicted failure load within ~1%.
- FEA corroborated stress distributions and failure location.
- Deflection discrepancies are explained by anisotropy, hygroscopic behavior, and boundary compliance.
- Stress-based design remains reliable for additively manufactured polymers when assumptions are clearly stated.

9. References & Appendices

- Appendix A: Engineering Notebook – Stress Analysis

Pg. 1

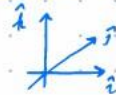
Jimmy Trozaw
12/22/25

3-D Printed Polylactic Acid (PLA) Batic – Bambu's Technical Data Specs (TDS)

TDS indicates the polymeric material to be anisotropic.

Young's Modulus	Tensile Strength	Bending Strength
$E(x-y) = 2580 \pm 220 \text{ MPa}$	$\sigma_{UTS}(x-y) = 35 \pm 4 \text{ MPa}$	$\sigma_b(x-y) = 76 \pm 5 \text{ MPa}$
$E(z) = 2060 \pm 170 \text{ MPa}$	$\sigma_{UTS}(z) = 31 \pm 3 \text{ MPa}$	$\sigma_b(z) = 59 \pm 6 \text{ MPa}$

* Note: The z -values indicate that the PLA holds weaker bonds in the $\hat{k}$ direction in comparison to the $\hat{i}$ & $\hat{j}$.



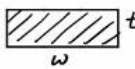
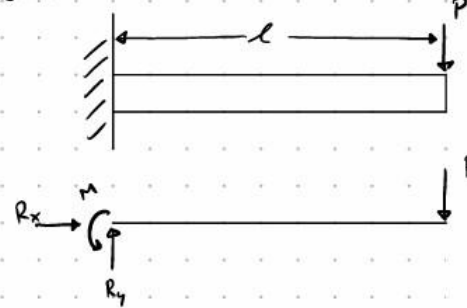
Due to inadequate data to compute anisotropic stress, treating this mechanical properties of PLA as an effective isotropic with conservative averaged properties:

Young's Modulus - Low Band	Tensile Stress - Low Band	Bending Stress - Low Band
$E(x-y) = 2360 \text{ MPa}$	$\sigma_{UTS}(x-y) = 31 \text{ MPa}$	$\sigma_b(x-y) = 71 \text{ MPa}$
$E(z) = 1890 \text{ MPa}$	$\sigma_{UTS}(z) = 28 \text{ MPa}$	$\sigma_b(z) = 53 \text{ MPa}$
$E_{150} \approx 2000 \text{ MPa}$	$\sigma_{UTS,150} \approx 30 \text{ MPa}$	$\sigma_{b,150} \approx 60 \text{ MPa}$

* Note: Estimated the effective isotropic values to ^{be} somewhat conservative since the high bands are the best-scenario to which we need to adjust them slightly lower than the high-band but slightly higher than the low-band, compensating the withdrawal stress against unforeseen, real-world factors.

Pg. 2

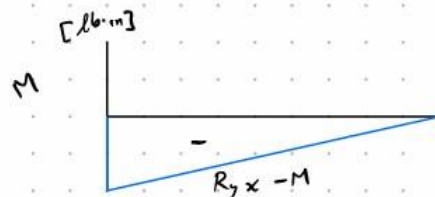
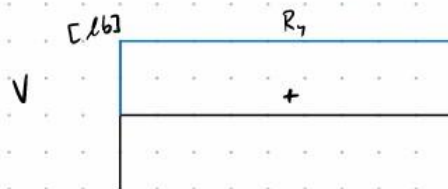
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Load Analysis

$$\rightarrow \sum F_x = 0 \therefore R_x = 0$$

$$\uparrow \sum F_y = 0 \therefore R_y = P \uparrow \leftarrow \text{P input value}$$

$$\circlearrowleft \sum M = 0 \therefore M - Pl = 0 \Rightarrow M = Pl \text{ ccw}$$

Singularity Function:

$$q(x) = M \langle x \rangle^{-2} + R \langle x \rangle^{-1} - P \langle x - l \rangle^{-1}$$

$$V(x) = \int q(x) dx = M \langle x \rangle^{-1} + R \langle x \rangle^0 - P \langle x - l \rangle^0$$

$$M(x) = \int V(x) dx = -M \langle x \rangle^0 + R \langle x \rangle^1 - P \langle x - l \rangle^1$$

Tip Deflection Analysis (Euler-Bernoulli)

$$EI \frac{d^4 y}{dx^4} = \int M(x) dx$$

$$= -M \langle x \rangle^1 + \frac{R}{2} \langle x \rangle^2 - \frac{P}{2} \langle x - l \rangle^2 + C_1$$

$$EI \delta = \iint M(x) dx$$

$$= -\frac{M}{2} \langle x \rangle^2 + \frac{R}{6} \langle x \rangle^3 - \frac{P}{6} \langle x - l \rangle^3 + C_1 x + C_2$$

where boundary condition: $x=0, y=0 \Rightarrow C_1 = C_2 = 0$

$$\therefore \begin{cases} \text{Slope: } \theta = \frac{1}{EI} [-M \langle x \rangle + \frac{R}{2} \langle x \rangle^2] \\ \text{Deflection: } \delta = \frac{1}{EI} [-\frac{M}{2} \langle x \rangle^2 + \frac{R}{6} \langle x \rangle^3] \end{cases} \leftarrow \text{P value}$$

* Note: The load P value input is needed to calculate those analysis of load & deflection numerically. To obtain the P value, performing the stress analysis is needed (Pg. 3).

Stress Analysis

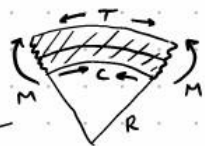
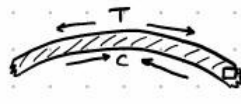
Bending stress: $\sigma_b = \frac{M c}{I}$, $I = \frac{w t^3}{12}$

$$= \frac{(P \times l) \frac{t}{2}}{w t^3 / 12} \therefore \sigma_b = 6 \frac{P \times l}{w t^2}$$

$$\Rightarrow P = \frac{\sigma_b w t^2}{6 l}, \text{ where } \sigma \text{ is the governing stress}$$

$$\therefore \sigma = \sigma_{\text{Tensile}}$$

Selecting the governing stress:



$$\sigma_{UTS} < \sigma_b$$

$\therefore \sigma_{UTS, 150}$ is the governor. ✓

$$\sigma_{UTS, 150} \approx 30 \text{ MPa} \left(\frac{145 \text{ psi}}{1 \text{ MPa}} \right)$$

$$\therefore \sigma_{UTS, 150} \approx 4350 \text{ psi}$$



$$\Rightarrow \tau_{max} = \frac{3}{2} \frac{V}{A} = \frac{3}{2} \frac{P}{w t}$$

$$\frac{\tau_{max}}{\sigma_{max}} = \frac{\frac{3}{2} \frac{P}{w t}}{6 \frac{P \times l}{w t^2}} \therefore \frac{\tau_{max}}{\sigma_{max}} = \frac{t}{4 l}$$

$$\left[\frac{\tau_{max}}{\sigma_{max}} = \frac{0.375}{4(6.5)} \approx 0.014 \right]$$

* Note: Failure mode is governed by the utilization ratio. This case with 7" x 1" x 0.375" beam will experience shear stress only by ~1.4%. Majority is the bending stress, attributed by the tensile stress at the outer fibers of the beam.

Failure Mode ($\sigma = \sigma_{\text{tens}, 150}$):

$$P_{\text{fail}} = \frac{\sigma_{\text{tens}} w t^2}{6 l} = \frac{4350 \frac{\text{lb}}{\text{in}^2} (1 \text{ in}) (0.375 \text{ in})^2}{6 (6.5 \text{ in})}$$

$$\therefore P_{\text{fail}} = 15.69 \text{ lb}$$

Loading Safety Design (Factor of Safety):

$$\sigma_{\text{allow}} = \frac{\sigma_{\text{tens}}}{n} \equiv P_{\text{allow}} = \frac{P_{\text{fail}}}{n}$$

$$n = 2: P_{\text{allow}} = \frac{P_{\text{fail}}}{2} \therefore P_{\text{allow}, 2} \approx 7.85 \text{ lb}$$

$$n = 2.5: P_{\text{allow}} = \frac{P_{\text{fail}}}{2.5} \therefore P_{\text{allow}, 2.5} \approx 6.28 \text{ lb}$$

$$n = 3: P_{\text{allow}} = \frac{P_{\text{fail}}}{3} \therefore P_{\text{allow}, 3} \approx 5.23 \text{ lb}$$

* Note: Substitute those values of P_{fail} , $P_{\text{allow}, 2}$, $P_{\text{allow}, 2.5}$, and $P_{\text{allow}, 3}$ into equations in load & deflection analysis (Pg. 2) to get the numerical results. This is done via MATLAB.

Failure Theory:

Distortion Energy (Von-Mises): $\sigma' = \sqrt{\sigma^2 + 3\tau_{\text{max}}^2}$, $\tau_{\text{max}} = \frac{3}{2} \frac{P}{A} = \frac{3 (15.69) \text{ lb}}{2 (1 \times 0.375) \text{ in}^2}$

$$\tau_{\text{max}} \approx 62.76 \text{ psi}$$

$4350 \text{ psi} (0.014) \approx 61 \text{ psi} \checkmark$

$$= \sqrt{4350^2 + 3(62.76)^2} \text{ psi}$$

$$\sigma' = 4350.5 \text{ psi}$$

Max. Principal Stress: $\sigma_{1,2} = \frac{\sigma}{2} \pm \sqrt{\left(\frac{\sigma}{2}\right)^2 + \tau^2} \Rightarrow 2175 \pm \sqrt{(2175)^2 + (62.76)^2}$

$$\sigma_{1,2} = 2175 \pm 2176 \text{ psi}$$

$$\therefore \sigma_1 = 4351 \text{ psi}, \sigma_2 = \sigma_3 = 0$$

* Note: Uniaxial bending stress dominates. Again, the shear stress is significantly lower than the normal stress ($\sim 1.4\%$, pg. 3) to which we can disregard the shear stress in the failure mode calculation.

Pg. 5

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Theoretical Results

FoS	P [lb]	δ [in]
3	5.23	0.376
2.5	6.28	0.451
2	7.85	0.563
1	15.69	1.127

Experimental Result

FoS	P [lb]	δ [in]
3	5.25	0.471
2.5	6.28	0.586
2	7.81	0.760
1	15.55	N/A

Percentage Error

$$\frac{\text{Exp.} - \text{The.}}{\text{The.}} (100\%)$$

FoS	E% (δ)
3	25.3%
2.5	29.9%
2	35.0%
1	N/A

E% (P_{fail})
0.87%

* Note: The deflection error percentage is large may be attributed by a several root causes: Boundary conditions realism (clamp is not perfect fixed support, such as micro-rotation when loaded) that is not accurately adheres to the Euler-Bernoulli Theory, PLA holds hygroscopic behavior that experiences reduced modulus & strength, and PLA is anisotropic, which is not as same as effective isotropic that is being conducted with conservative values in this calculation.

* Note: FEA indicates the maximum principal stress of 4300.5 psi @ the fixed end with the similar boundary conditions, resulting in $\sim 1\%$ of error. FEA also shows that distortion energy theory (Von Mises) is much lower than the normal stress by $\sim 1.05\%$, aligning to our expectation. Also, the experiment shows that the failure load is much close to the theoretical value of failure load within $\sim 0.87\%$ error despite of the deflection, as I do not have proper experimental equipment, such as longer than a 1 inch plunger dial indicator, a high-quality clamped, and dried PLA filament.

- Appendix B: MATLAB Live Script

```

1 % Cantilever Beam Project - PLA (Bambu) - Analytical Model
2 % Units: inches, lbf, psi
3 % Purpose: Analysis of Stresses and Deflection - Cantilever Beam with Concentrated Load
4
5 clear; clc;
6
7 % -----
8 % 1) Inputs
9 % -----
10
11 L = 6.5; % beam length [in]
12 w = 1.0; % beam width [in]
13 t = 0.375; % beam thickness [in]
14 c = t / 2; % distance from neutral axis to outer fiber [in]
15 E = 290e3; % Young's modulus [psi]
16 A = w * t; % cross-sectional area [in^2]
17 I = (w * t^3) / 12; % moment of inertia [in^4]
18 sigma_ult = 4350; % ultimate tensile strength [psi]
19
20 % -----
21 % 2) Vectors of Load, Stresses, and Deflection
22 % -----
23
24 P_max = 15.69; % max load to plot [lbf] - obtained from hand calculation
25 nPts = 400; % number of points in load sweep
26 P = linspace(0, P_max, nPts); % tip load [lbf]
27 M = P * L; % bending moment at fixed end [lbf-in]
28 sigma_bend = (M * c) / I; % Bending stress on outer fiber at fixed end [psi]
29 tau_max = (3 * P) ./ (2 * A); % Max transverse shear stress for rectangular section [psi]
30
31 % Tip deflection (Euler-Bernoulli, concentrated load at free end):
32 delta_tip = (E * I)^-1 * (-M/2 * L^2 + P/6 * L^3); % [in]
33
34 % -----
35 % 3) Get user's Factor of Safety value
36 % -----
37
38 N = input("Enter the required factor of safety for this design: ");
39
40 % -----
41 % 5) Failure load
42 % -----
43
44 P_fail = (sigma_ult * w * t^2) / (6 * L); % [lbf]
45 delta_fail = (E * I)^-1 * (-(P_fail * L)/2 * L^2 + P_fail/6 * L^3); % [in]
46 sigma_fail = (P_fail * L * c) / I; % [psi]
47 tau_fail = (3 * P_fail) / (2 * w * t); % [psi]
48 ratio_tau_sigma = tau_fail / sigma_fail; % quick sanity check, unitless
49
50 % -----
51 % 6) Allowable load with Factor of Safety, N (scalar result)
52 % -----
53
54 P_allow = P_fail / N; % [lbf]
55 M_allow = P_allow * L; % [lbf-in]
56 delta_allow = (E * I)^-1 * (-M_allow/2 * L^2 + P_allow/6 * L^3);
57 sigma_allow = sigma_fail / N;
58 tau_allow = tau_fail / N;
59
60 % -----
61 % 7) Print key results
62 % -----
63
64 fprintf("<strong>=== Cantilever Beam Analytical Summary (PLA) ===");
65 fprintf("Geometry: L = %.2f in, w = %.2f in, t = %.2f in", L, w, t);
66 fprintf("Section: I = %.6f in^4, A = %.3f in^2", I, A);
67 fprintf("Material: E = %.0f psi, sigma = %.0f psi", E, sigma_ult);
68 fprintf("Theoretical failure load (sigma_{ultimate}): <strong>P_fail = %.2f lbf", P_fail);
69 fprintf("Theoretical maximum deflection: <strong>delta_{fail} = %.4f inch", delta_fail);
70
71 fprintf("\n\n<strong>=== Deflection at Allowable Load at Factor of Safety, N = %.2f ===", N);
72 fprintf("Recommended allowable load: <strong>P_{allow} = %.2f lbf", P_allow);
73 fprintf("Predicted tip deflection at P_allow: <strong>delta_{allow} = %.4f in", delta_allow);
74 fprintf("Allowable normal stress: <strong>sigma_{allow} = 2%.2f psi", sigma_allow);
75

```

```

76 fprintf("\n \n<strong>=== Quick Sanity Check @ P_fail = %.2f lb ===", P_fail);
77 fprintf("    sigma_bend = %.0f psi (ultimate tensile at %.0f psi)", sigma_fail, sigma_ult);
78 fprintf("    tau_max = %.0f psi", tau_fail);
79 fprintf("    tau_max / sigma_bend = %.3f", tau_fail / sigma_fail);
80 if ratio_tau_sigma < 0.5
81     fprintf("The Tresca failure criterion may be considered negligible because the shear-to-normal stress ratio is below 0.5.");
82 else
83     fprintf("The shear-to-normal stress ratio exceeds 0.5, and the Tresca failure criterion should be applied.");
84 end
85
86 % -----
87 % 8) Plots
88 % -----
89
90 figure;
91 hold on;
92 plot(P, sigma_bend, "LineWidth", 1.5);
93 title("Cantilever Beam: Bending Stress vs Load");
94 grid on;
95 xlabel("Load P [lbf]");
96 ylabel("Max Bending Stress \sigma [psi]");
97 yline(sigma_ult, "--", "LineWidth", 1.5);
98 xline(P_fail, "--", "LineWidth", 1.5, "Color", "r");
99 if N > 1
100     yline(sigma_allow, "--", "LineWidth", 1.5, "Color", "m");
101     xline(P_allow, "--", "LineWidth", 1.5, "Color", "b");
102     legend("", "\sigma_{ultimate}", "P_{fail}", "\sigma_{allow}", "P_{allow}");
103 else
104     legend("", "Ultimate Strength", "P_{fail}");
105 end
106 xlim([0 30]);
107 ylim([0 5000]);
108 hold off;
109
110 figure;
111 hold on;
112 plot(P, tau_max, "LineWidth", 1.5);
113 title("Cantilever Beam: Shear Stress vs Load");
114 grid on;
115 xlabel("Load P [lbf]");
116 ylabel("Max Shear Stress \tau [psi]");
117 yline(tau_fail, "--", "LineWidth", 1.5);
118 xline(P_fail, "--", "LineWidth", 1.5, "Color", "r");
119 if N > 1
120     yline(tau_allow, "--", "LineWidth", 1.5, "Color", "m");
121     xline(P_allow, "--", "LineWidth", 1.5, "Color", "b");
122     legend("", "\tau_{fail}", "P_{fail}", "\tau_{allow}", "P_{allow}");
123 else
124     legend("", "\tau_{fail}", "P_{fail}");
125 end
126 xlim([0 25]);
127 ylim([0 70]);
128 hold off;
129
130 figure;
131 hold on;
132 plot(P, delta_tip, 'LineWidth', 1.5);
133 title('Cantilever Beam: Tip Deflection vs Load');
134 grid on;
135 xlabel('Load P [lbf]');
136 ylabel('Tip Deflection \delta [in]');
137 yline(delta_fail, "--", "LineWidth", 1.5);
138 xline(P_fail, "--", "LineWidth", 1.5, "Color", "r");
139 if N > 1
140     yline(delta_allow, "--", "LineWidth", 1.5, "Color", "m");
141     xline(P_allow, "--", 'LineWidth', 1.5, "Color", "b");
142     legend("", "\delta_{fail}", "\delta_{allow}", "P_{fail}", "P_{allow}");
143 else
144     legend("", "\delta_{fail}", "P_{fail}");
145 end
146 xlim([0 18]);
147 ylim([-1.3 0]);
148 hold off;

```

- **Appendix C: Safety Factor Studies**

Factor of Safety: 1.00

=== Cantilever Beam Analytical Summary (PLA) ===

Geometry: L = 6.50 in, w = 1.00 in, t = 0.38 in

Section: I = 0.004395 in⁴, A = 0.375 in²

Material: E = 290000 psi, sigma = 4350 psi

Theoretical bending failure load (sigma_{ultimate}): P_fail = 15.69 lbf

Theoretical maximum deflection: delta_{fail} = -1.1267 inch

=== Deflection at Allowable Load at Factor of Safety, N = 1.00 ===

Recommended allowable load: P_{allow} = 15.69 lbf

Predicted tip deflection at P_{allow}: delta_{allow} = -1.1267 in

Allowable normal stress: sigma_{allow} = 24350.00 psi

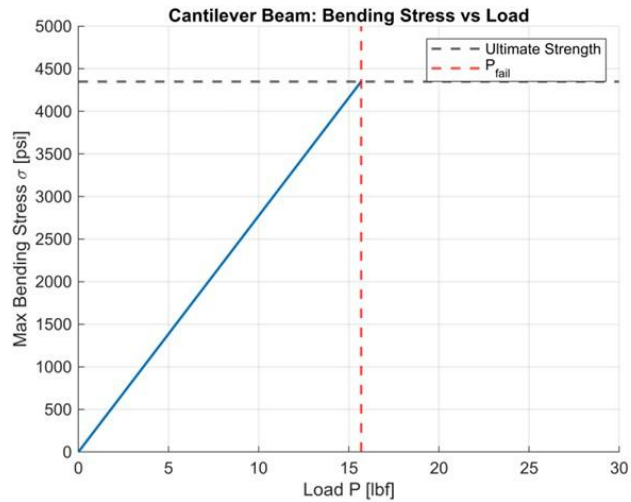
=== Quick Sanity Check @ P_fail = 15.69 lb ===

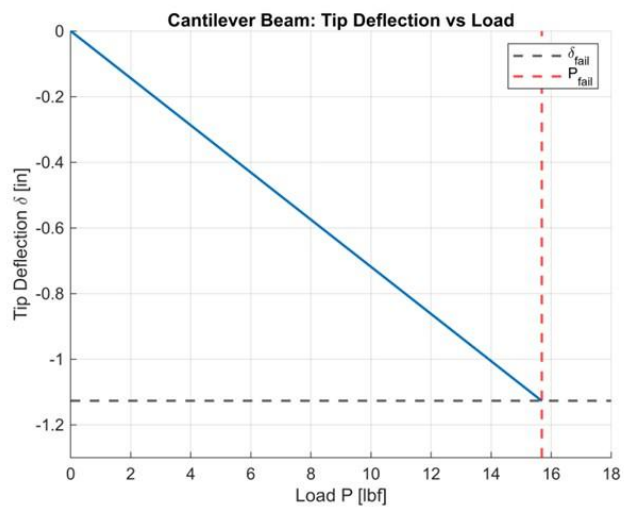
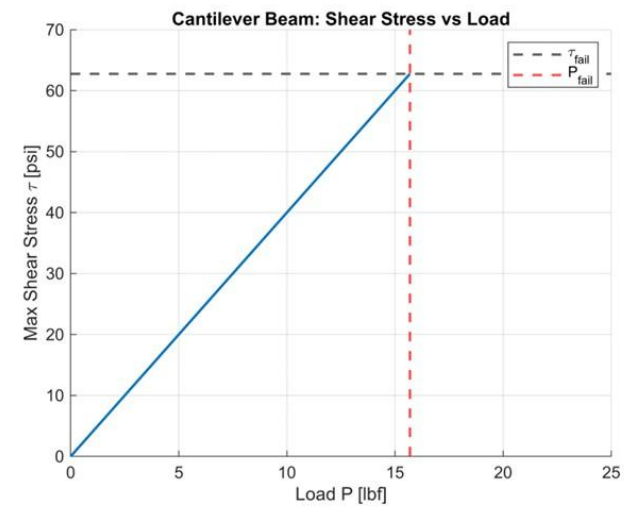
sigma_bend = 4350 psi (ultimate tensile at 4350 psi)

tau_max = 63 psi

tau_max / sigma_bend = 0.014

The Tresca failure criterion may be considered negligible because the shear-to-normal stress ratio is below 0.5.





Factor of Safety: 2.00

=== Cantilever Beam Analytical Summary (PLA) ===

Geometry: $L = 6.50$ in, $w = 1.00$ in, $t = 0.38$ in

Section: $I = 0.004395$ in⁴, $A = 0.375$ in²

Material: $E = 290000$ psi, $\sigma_{\text{ultimate}} = 4350$ psi

Theoretical bending failure load (σ_{ultimate}): $P_{\text{fail}} = 15.69$ lbf

Theoretical maximum deflection: $\delta_{\text{fail}} = -1.1267$ inch

=== Deflection at Allowable Load at Factor of Safety, $N = 2.00$ ===

Recommended allowable load: $P_{\text{allow}} = 7.84$ lbf

Predicted tip deflection at P_{allow} : $\delta_{\text{allow}} = -0.5633$ in

Allowable normal stress: $\sigma_{\text{allow}} = 22175.00$ psi

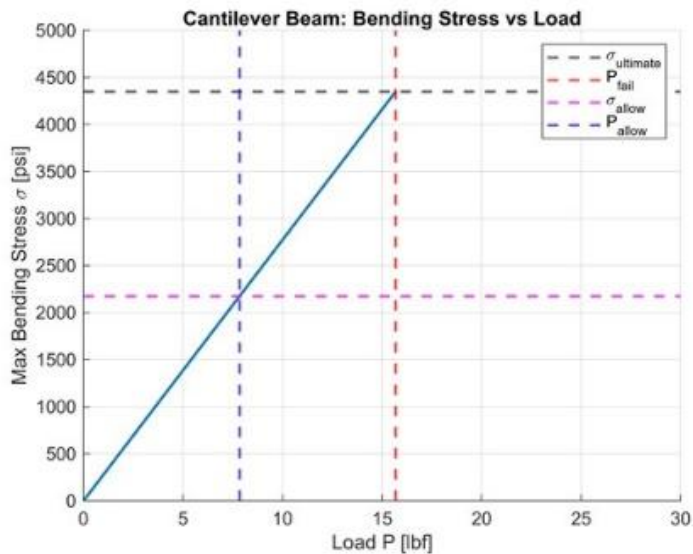
=== Quick Sanity Check @ $P_{\text{fail}} = 15.69$ lb ===

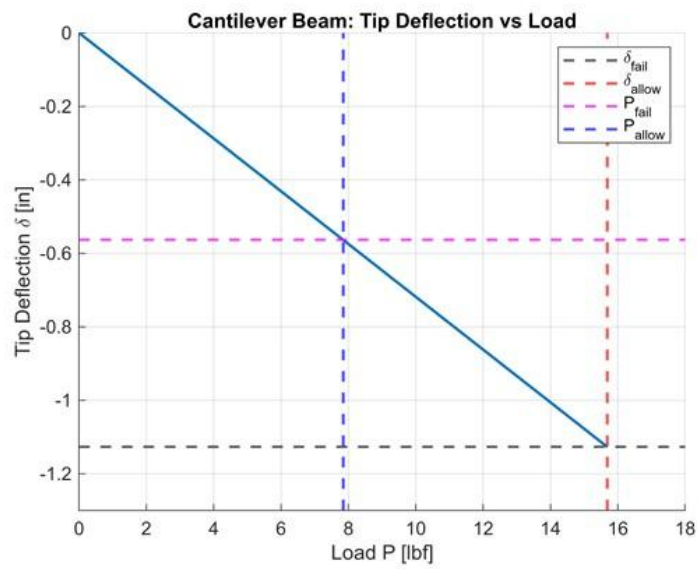
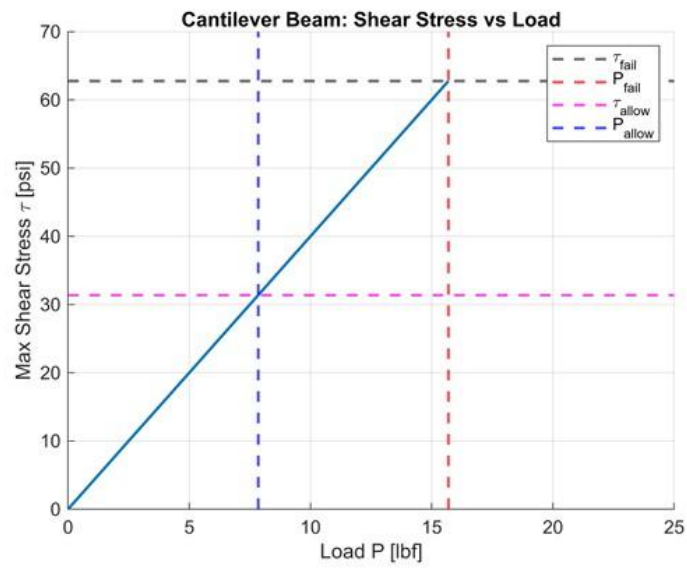
$\sigma_{\text{bend}} = 4350$ psi (ultimate tensile at 4350 psi)

$\tau_{\text{max}} = 63$ psi

$\tau_{\text{max}} / \sigma_{\text{bend}} = 0.014$

The Tresca failure criterion may be considered negligible because the shear-to-normal stress ratio is below 0.5.





- **Appendix D: Digital Force Data**

